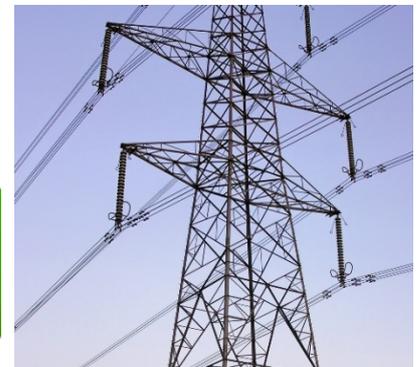
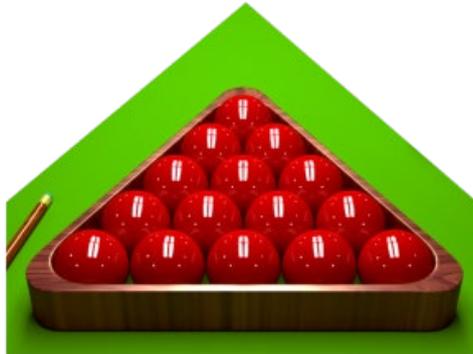




Mysterious Maths



Hands-on activities bringing the
mysteries of maths to life

Curriculum for Excellence

Mysterious Maths is a set of activities to explore the **mystery** of maths. Designed for learners in the second level, the activities could support the following Outcomes in Mathematics and Technologies.

The activities can be delivered within an extra-curricular club, or within the curriculum, to suit the school. Suppliers for resources are suggested where required and a set of templates is provided.

We would be pleased to hear your feedback or suggestions: yesc@scdi.org.uk

Activity 1 – Tiling	
<p>I have explored simple 3D objects and 2D shapes and can identify, name and describe their features using appropriate vocabulary. MTH 1-16a</p> <p>I can explore and discuss how and why different shapes fit together and create a tiling pattern with them. MTH 1-16b</p>	<p>Having explored a range of 3D objects and 2D shapes, I can use mathematical language to describe their properties, and through investigation can discuss where and why particular shapes are used in the environment. MTH 2-16a</p> <p>When exploring technologies in the world around me, I can use what I learn to help to design or improve my ideas or products. TCH 2-01a</p>
Activity 2 – Counters	
<p>I have spotted and explored patterns in my own and the wider environment and can copy and continue these and create my own patterns. MTH 0-13a</p> <p>Through exploring number patterns, I can recognise and continue simple number sequences and can explain the rule I have applied. MTH 1-13b</p>	<p>Having explored more complex number sequences, including well-known named number patterns, I can explain the rule used to generate the sequence, and apply it to extend the pattern. MTH 2-13a</p>
Activity 3 – Folding	
<p>I have had fun creating a range of symmetrical pictures and patterns using a range of media. MTH 0-19a</p> <p>I have explored symmetry in my own and the wider environment and can create and recognise symmetrical pictures, patterns and shapes. MTH 1-19a</p>	<p>I can illustrate the lines of symmetry for a range of 2D shapes and apply my understanding to create and complete symmetrical pictures and patterns. MTH 2-19a</p>

Careers

Maths is a key skill in many jobs so we have included some links for your club or class to learn more about career options in a range of STEM disciplines.

Skills Development Scotland's [My World of Work](#) is a great website with information and activities to support young people to find a career to suit their skills and interests. Case studies, videos, and a wealth of information for careers in all sectors including a [data-analyst](#) and [statistician](#).

[Maths Careers](#) has a host of information and resources. This [article](#) about the use of maths in getting a job highlights a range of careers which use maths from designing computer games, medical research, meteorology, car design; proving maths is not just for mathematicians!

If you would like an engineer to visit your school, you can request a STEM Ambassador from your local [STEMNET contract holder](#). STEM Ambassadors are inspiring role models who volunteer to share their expertise and enthusiasm for Science, Technology, Engineering and Maths with local schools.

Objectives

Maths is a game with really simple rules; a game which inadvertently (or *mysteriously*) reveals something about **why** the world is the way it is. Maths is fun, beautiful, and amazingly (or *mysteriously*) useful.

Purpose

Noticing and studying patterns allows us to go beyond learning by experience. For example, to know that you can build a large structure by combining all your building blocks even though you have previously only made smaller structures, or that by saving pocket money you can purchase something more expensive.

Maths allows engineers to know that something will work *before* it has been made: to *know* that a new bridge can stand and hold the weight of traffic or to *know* that the International Space Station can orbit the Earth safely. Maths lets us understand the world, and to therefore exploit its opportunities.

Method

The three activities; **Tiling**, **Counters** and **Folding**, allow pupils to discover and experience maths hands-on, to literally get a 'feel' for maths. They can be completed individually or as a set, to reinforce the theme of *Mystery*.

With preparation of common materials and a set of templates (provided on pages 12 - 29), the essentials of each activity can be completed in 15 minutes. Extension activities can extend this time to feed curiosity.

- At the start of each activity, emphasise that **we** are making up the rules of the game, and that the rules are very simple.
- During the activities, take opportunities to point out the patterns which '*mysteriously*' emerge from these simple rules – patterns which are not dictated by us, but by the world.
- Finish each activity with real-world examples of the mathematical patterns which have been discovered.

Activity 1 – Tiling

Preparation:

Make colour prints of the templates of triangles, squares, pentagons, hexagons; and one B&W fish template for each group. Ideally use 120gsm paper. Cut out one set of shapes (not the fish) for each group of 2-4 children. The shapes can be re-used if stored in an envelope. A clear CD sleeve is ideal. You also need: Glue sticks, scissors, felt pens, flipchart or large sheet of paper, a football, and an [image of bees on a honeycomb](#).

Using the large pentagon template to make two halves of a 'dodecahedron' from thick cardboard. This takes longer than you would think and might be a project for older pupils. No-one needs to remember the word 'dodecahedron' – a solid shape made from 12 pentagons.

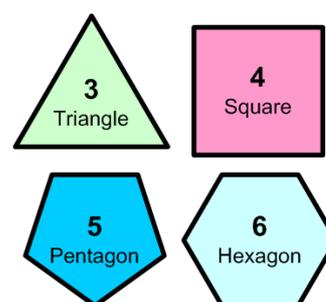
- 100 sheets 120gsm A4 by Craft, £3.50 [Amazon](#)
- 100 CD sleeves, £3.99: [Amazon](#)

Explain the simple rule which we are going to use:

Make regular shapes with 3, 4, 5, and 6 sides.

To keep things really simple, make all sides the same length.

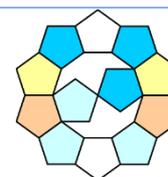
Show the shapes, and ask who knows the correct names:
Triangle, square, pentagon, hexagon.



Divide into groups of 2-4, give each group a set of previously cut shapes and ask them to imagine they are tiling a bathroom floor. They are to fit the tiles together, with no gaps which would let water leak through. A pretty pattern is good, but it is essential to have no gaps.

Keep the finished pattern to show the rest of the class.

[Ensure the pentagon group doesn't give up, but preserves a 'failed' attempt, to demonstrate that pentagons will NOT tile without gaps.]



After 5 minutes, get each group ready to show everyone their patterns. Admire the prettiest, congratulate the neatest, but draw everyone's attention to the fact that, although tiling works with shapes which have 3, 4, and 6 sides, **5 sides don't work** – there are always gaps! Although **we** made up the rules of the game, we didn't decide this – the world did. Gather the shapes for re-use.

Another example of a polygon



An experiment to test the theory!

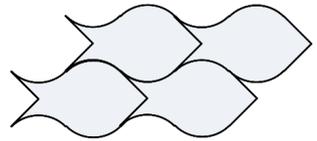
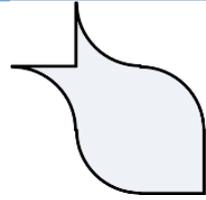
Discuss other shapes which might, or might not, tile without gaps (no wrong answers). Show the fish shape, guess whether this will work.

To find out, give each group a sheet of fish and cut out one fish each, as accurately as possible. (This is Science – first a guess, then an experiment test the theory.)

Add their initials; those finished first can add an eye or scales....

Taking great care to align the first 4 shapes, get everyone to glue their fish onto a flip chart or similar to make a class 'tessellation' with no gaps.

The conclusion (which should be a bit surprising or *mysterious*) is that tiling **does** work with fish; so it's not just regular shapes which can tile together.



Another amazing demonstration!

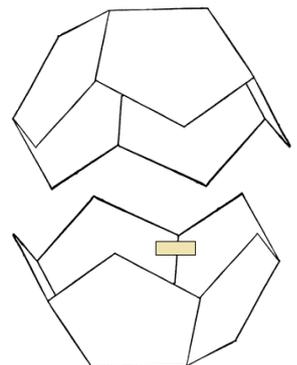
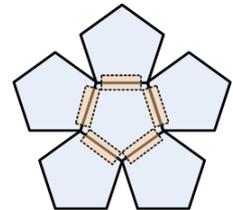
Place half the dodecahedron (i.e. 6 pentagons taped together) flat on a table, and remind everyone that we know that these Pentagons are 'awkward' shapes - which don't fit together.

Point out the gaps between the pentagons, but now watch this...

... lift up the side pentagons, and hold them in place with masking tape to produce a solid shape with no gaps. Drop the other half (prepared earlier) on top. In 3 dimensions, the awkward ugly ducklings become perfectly fitting swans!

Who would have guessed? Or, ...how *mysterious* is that?

(Incidentally, hexagons *will not* fit together in 3D).



Look at a real football, ask what shapes are tiled together. Look carefully (more carefully!), and trace out *both* hexagons and pentagons – so this tiling game turns out to be relevant to football.

Finally, look at a large picture of a honeycomb, note the hexagons fitting neatly together, and ponder the *mystery* of how bees 'know' that pentagons don't work.



Activity 1 – Tiling extension activities

Structures

Make 'regular' polygons from pencils joined with elastic bands.
Make polygons with 3, 4, 5 sides.

Note that the triangle is stable, but other shapes like squares are floppy.

Reflect that, once again, **we** did not decide this; the shapes 'decided' it for us. Why the triangle is stable is **mysterious**.

Look at pictures of lattice girders (e.g. [electricity pylon](#)), and spot how engineers use triangles to make stable structures.

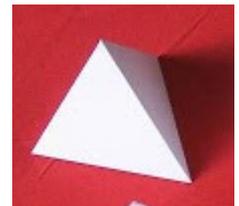
Take a floppy square of pencils; add a diagonal brace to split it into two triangles. 2 triangles = 1 square.

That is **Engineering**: using maths to make useful things



Paper Tetrahedron [A solid with 4 faces: *Tetra* = four; *hedra* = face]

Hand out copies of the tetrahedron template.



Cut carefully around the outline and fold/unfold accurately along each line.
Use a glue stick on the tabs to make a tetrahedron solid.
You could decorate and hang up for any theme!

Dice

Ask what shape dice are.

Point out that it does not have to be like that! Maths helps you to think 'outside the box'. Play with the two Polyhedral dice provided in your kit.
More available for £5.99: [Amazon](#)

These regular solids result from 'tiling' regular polygons in three dimensions. This only works with triangles, squares and pentagons, and (**mysteriously**) only five solid shapes are possible: tetrahedron (4 triangles), cube, octahedron, dodecahedron (12 pentagons) and icosahedron (20 triangles).



In 2D, as we saw, the only regular shapes which tile have 3, 4 and 6 sides- but you could try a mixture of shapes for more fun!

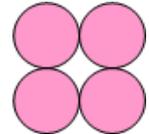
Activity 2 – Counters

Preparation:

Make a number line (1 to 59) - from A3 copies of the template provided. Fix on a wall at child head height. Cut red squares and blue triangles from the A4 template provided, with small Blu-tac dots behind. Use plastic counters or 1p pieces (500 does a class of 30), in a paper cup, for each group. Also need: 10 unpatterned paper plates, with large Blu-tac dots behind.

Stick four paper plates on the wall in a square, touching each other.

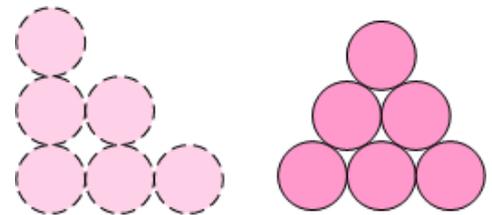
Explain that we are going to look at the **shape** of a number.



Get someone to identify this shape as a square, and explain that in this sense, **4 is a square number**.

Repeat by making a triangle with 6 more paper plates, discovering that **6 is a triangular number**.

(Either of these arrangements can be used, but the right hand 'pile of cannon balls' is the easiest to align.)

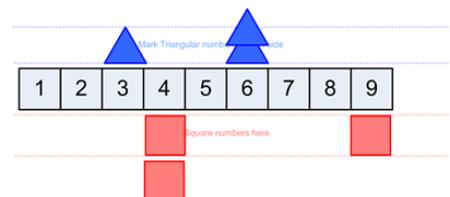


Divide into groups of 2-4. Show a cup of counters, and explain that each group has to arrange the counters to **discover** square numbers and triangular numbers. They will record these by placing markers (blue triangles or red squares) next the numbers on the number line. When they have found a number, they must show the counter arrangement to you for checking; if correct, you give them the coloured marker which they will place above or below the number line. Once everyone understands, give each group a cup of counters – about 20 for younger children, or 40 for older children. They can ask for more if they run out. This will soon develop into a race to see can find the most, or the biggest numbers.

Have a 'cheat-sheet' handy to help you check the right answers:

Square: 4, 9, 16, 25, 36, 49

Triangular: 3, 6, 10, 15, 21, 28, 36, 45, 55



More than one marker can be placed beside each number, to show agreement, and also so the slower groups do not get left out. Once enough numbers have been marked, collect in the coins and **review** what has been **discovered**. Move incorrect markers, add any missing ones.

First of all, note that although **we** invented the numbers, and made up the rules of this game, the numbers themselves determine which ones are square and which triangular. At the start we didn't know what would happen –it was a **mystery**. If we pick a bigger number ... say 86, we don't immediately know whether it will be triangular or not. (That is still a ... **mystery!**).

We can, however, now solve these **mysteries**:

- Which type of number is most common - square numbers, or triangular numbers?
- Is it possible for a number to be **both** square *and* triangular?

For a final flourish, ask what shape you get when adding two triangles together?
With a bit of prompting, agree that, **in the world of shapes**, two triangles can make a square.

Ask if that rule might work for numbers: can you make a square number by adding two triangular numbers? Try it:

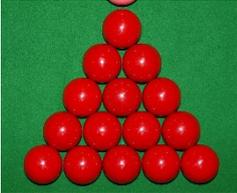
$$3 + 6 = 9$$

$$6 + 10 = 16$$

$$10 + 15 = 25$$

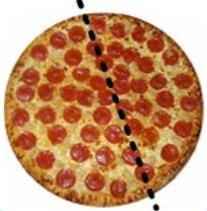
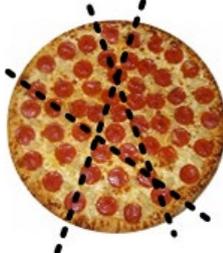
How's that for a **mystery**? This is an example of how mathematics sometimes gets ahead of its inventors, and can tell us the answers to questions we didn't think to ask.

Briefly look at some pictures, to show the relevance of number shapes in real life.

<p>Consider how many pins are used in 10 pin bowling.</p> <p>Look at a picture of ten bowling skittles.</p> <p>Is 10 a triangular number or a square number? Why does this shape of number work here? A mystery solved!</p>	 <p>Tesco Skittles</p>
<p>After 10, what is the next larger triangular number?</p> <p>What do you notice about the red balls in snooker?</p>	 <p>Snooker Reds <small>photo by barfisch under license CC-BY-SA 3.0</small></p>
<p>A square box of chocolates contains how many chocolates? A] 15 B] 16 C] 17</p> <p>In a shop or delivery van, square boxes stack together nicely without gaps. (A bit like Tiling).</p> <p>In a range of square boxes of different sizes, what numbers of chocolates are possible?</p>	 <p>neuhaustruffles</p>

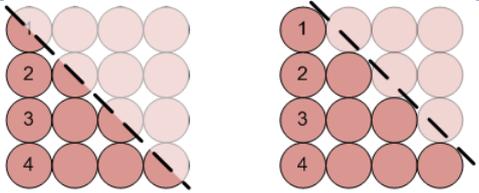
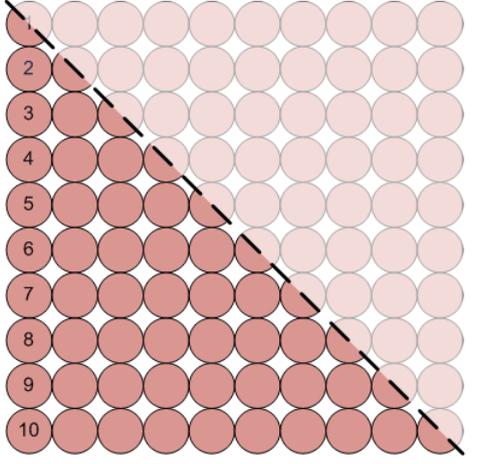
Activity 2 – Counters extension activities

Pizza Numbers

<p>Discover the Pizza Numbers and mark them on the number line.</p> <p>If you cut a circle (or pizza) with one straight line you get a maximum of two pieces.</p>	
<p>If you cut it with two straight lines you get a maximum of four pieces.</p>	
<p>How many pieces would you get if you cut it with three straight lines?</p> <p>In case you are wondering, the series is: 2, 4, 7, 11, 16, 22, 29, 37...</p>	
<p>A recent news article explains how to slice a pizza into perfectly sized slices: Popular Mechanics.</p>	

Gauss There is a story about the famous mathematician, Gauss, annoying his school teacher.

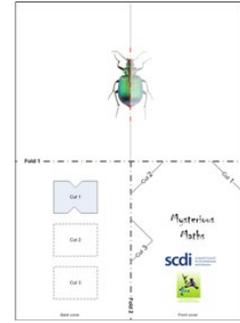
The teacher set the class the task of adding up all the numbers from 1 to 100, and settled down at his desk to get some peace. But a few minutes later, Gauss was finished and (annoyingly) had the right answer. He had spotted an easy way of doing this, using square and rectangular numbers.

<p>$1+2+3+4$ is a triangular number: the 4th triangular number.</p> <ul style="list-style-type: none"> ➤ This is close to half the square number $4 \times 4 = 16$ ➤ Square numbers are easy to count, by multiplying. ➤ Each half will have $16/2$ counters: i.e. 8. ➤ To get the 4th triangular number, we just have to add on the half counters along the diagonal. ➤ There are 4 half counters, so we need to add $4/2 = 2$. ➤ $8+2 = 10$; so the 4th triangular number is 10. 	
<p>$1+2+3+4+5+6+7+8+9+10$ is the 10th triangular number.</p> <ul style="list-style-type: none"> ➤ This is close to half the square number $10 \times 10 = 100$ ➤ Each half will have $100/2$ counters: i.e. 50. ➤ To get the 10th triangular number, we just have to add on the half counters along the diagonal. ➤ There are 10 half counters, so we need to add $10/2 = 5$ ➤ $50+5 = 55$; so the 10th triangular number is 55. 	
<p>Gauss spotted that the answer to the teacher's question was the 100th triangular number.</p> <ul style="list-style-type: none"> ➤ Which is ... _____? 	
<p>$1+2+3+4+...+100 =$ the 100th triangular number = 5050</p>	

Activity 3 – Folding

Preparation:

Print the 'Booklet' template on A4 paper per person.
Also need scissors, pencils, a glove, and felt pens.



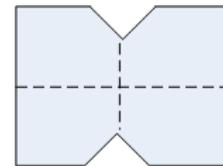
Fold the template twice, to make a booklet as shown.



Slowly open and close the booklet, and look at the picture of the beetle on the middle 'page'.
Discuss this as a reflection, or as an example of symmetry.
Note that since we folded the paper twice, we have made **two** reflections.

Now make **one** cut with scissors, to cut off the corner of the booklet along 'Cut 1'.

Open out the paper, and see the resulting shape.
See if you can spot how the single cut has become four cut edges.
It helps to colour the cut edges with a felt pen before opening.



On the back cover of the booklet, note that the top picture shows the shape of the paper after one cut.

Before making the next cut ('Cut 2'), draw the shape you expect the paper to be afterwards.

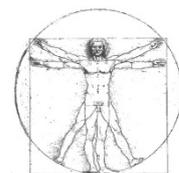
Now refold the booklet and cut off the second corner of the booklet, along 'Cut 2'. Open out the paper and see if you were right!

Savour the **mystery** of how **Cut 3** will turn out. It's the **mystery** which is important, not the answer!

Discussion: Remind your group that we have been looking at a **Reflection** (a fold), and that this is an example of a simple mathematical rule.

Note that plants and animals use this rule – for example, the beetle in the booklet, or a flower, or ...how about you? Hold out both arms; see that one is a **Reflection** of the other.

So **you** are living, growing, example of mathematics in action!



Point out that we have discovered that a complex shape can result from repeating a simple rule – we made **two** folds or Reflections. Repeating the rule **multiplies** the complexity: one fold doubles the paper, two folds gives *four* layers.

The natural world is full of amazing complexity which results from simple rules repeated over and over.

Look at some of the images.

[Flowers](#), [snowflakes](#), [shells](#)...

Wikimedia photo credits: Denis Barthelemy, Pen Waggener, Chris 73



Note that one outstretched arm is a 'copy' of the other. This principle is used in engineering. When designing (say) a car, if we make it symmetrical, then from one design we can get two parts of the car. In a similar way, the 'designer' of our bodies has got two arms for the price of one design!

To finish with a bit of **mystery**, put on one glove, and with two outstretched arms remind everyone that one arm is the same as the other one. So one glove is just a copy of the other one, yes?

No, not quite! Right and Left gloves are similar, but not quite the same. Shoes come in pairs, but a Right shoe isn't identical to a Left one. So a Reflection is copy, but not quite a copy - *Mysteriously* different!

Activity 3 – Folding extension activities

Snowflake

Print the snowflake template on A4 paper. Thin paper (50gsm) makes cutting easier.

- 500 sheets 50gsm A4 paper, £6.49: officesupplies.org.uk

A3 A4 A5 this needs only 1 plain sheet of A3 and 1 sheet of A4 per group.

Fold the A3 sheet in half, and discover that it has become A4 size - and that A4 has the same proportions as A3. Point out that it does not **have** to be like this: if, for example, paper was square, folding it once would not make another square. This is not a lucky chance, it is **engineering**! The shape $[1:\sqrt{2}]$ has been **designed** to be convenient and save waste. For example, an A4 booklet can be made by folding A3.

Fold again, and again, and note: **2** folds = **4** thicknesses, **3** folds= **8** thicknesses. See how many folds are possible before it gets too thick (about 7), and discover that this simple mathematical rule quickly and surprisingly (*Mysteriously*) gets out of hand when repeated.

Mysterious Maths Templates

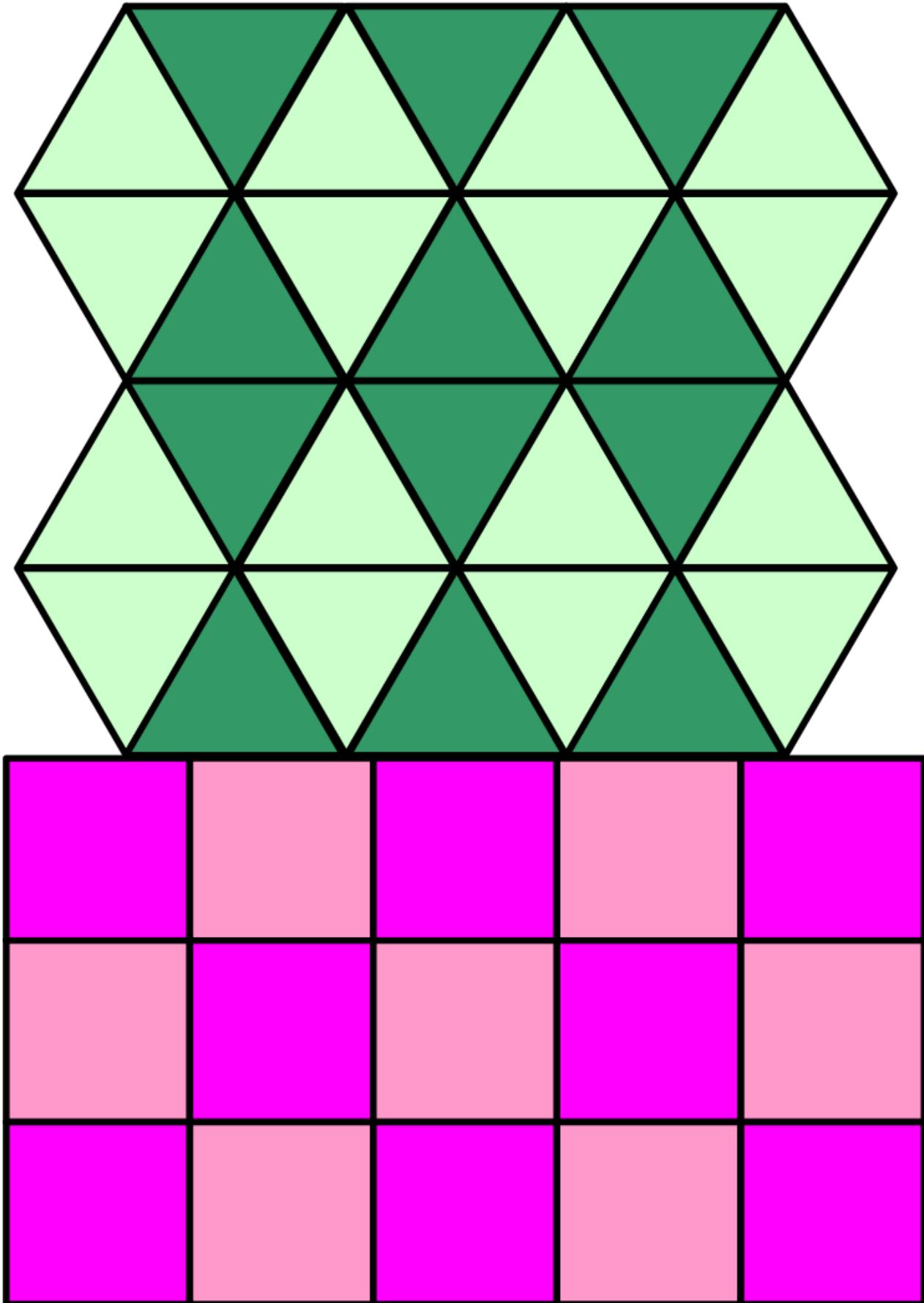
	Page	Size	Weight	Colour	Quantity
Activity 1 – Tiling					
Triangle & Square Tiles*	13	A4	120gsm	Colour	2
Pentagon & Hexagon Tiles*	15	A4	120gsm	Colour	2
Triangle & Square Tiles B&W	17	A4	120gsm coloured	B&W	2
Pentagon & Hexagon Tiles B&W	19	A4	120gsm coloured	B&W	2
Fish Tiles	21	A4	120gsm	B&W	1 per group
Dodecahedron* (cardboard pentagons)	23	A4	80gsm	B&W	1
Extension activity					
Tetrahedron	25	A4	80 or 120	B&W	1 between 2
Activity 2 – Counters					
Square & Triangle markers	31	A4	80gsm	Colour	1
Square & Triangle markers B&W	33	A4	80gsm coloured	B&W	2 – red & blue
Number line 1 to 9*	35	A3	80gsm	B&W	1
Number line 10 to 19*	37	A3	80gsm	B&W	1
Number line 20 to 29*	39	A3	80gsm	B&W	1
Number line 30 to 39*	41	A3	80gsm	B&W	1
Number line 40 to 49*	43	A3	80gsm	B&W	1
Number line 50 to 59*	45	A3	80gsm	B&W	1
Activity 3 – Folding					
A4 Fold & Cut booklet	27	A4	80gsm	B&W	1 each
Extension activity					
Snowflake	29	A4	50 or 80	B&W	1 each

*SHAPES PROVIDED IN PACK

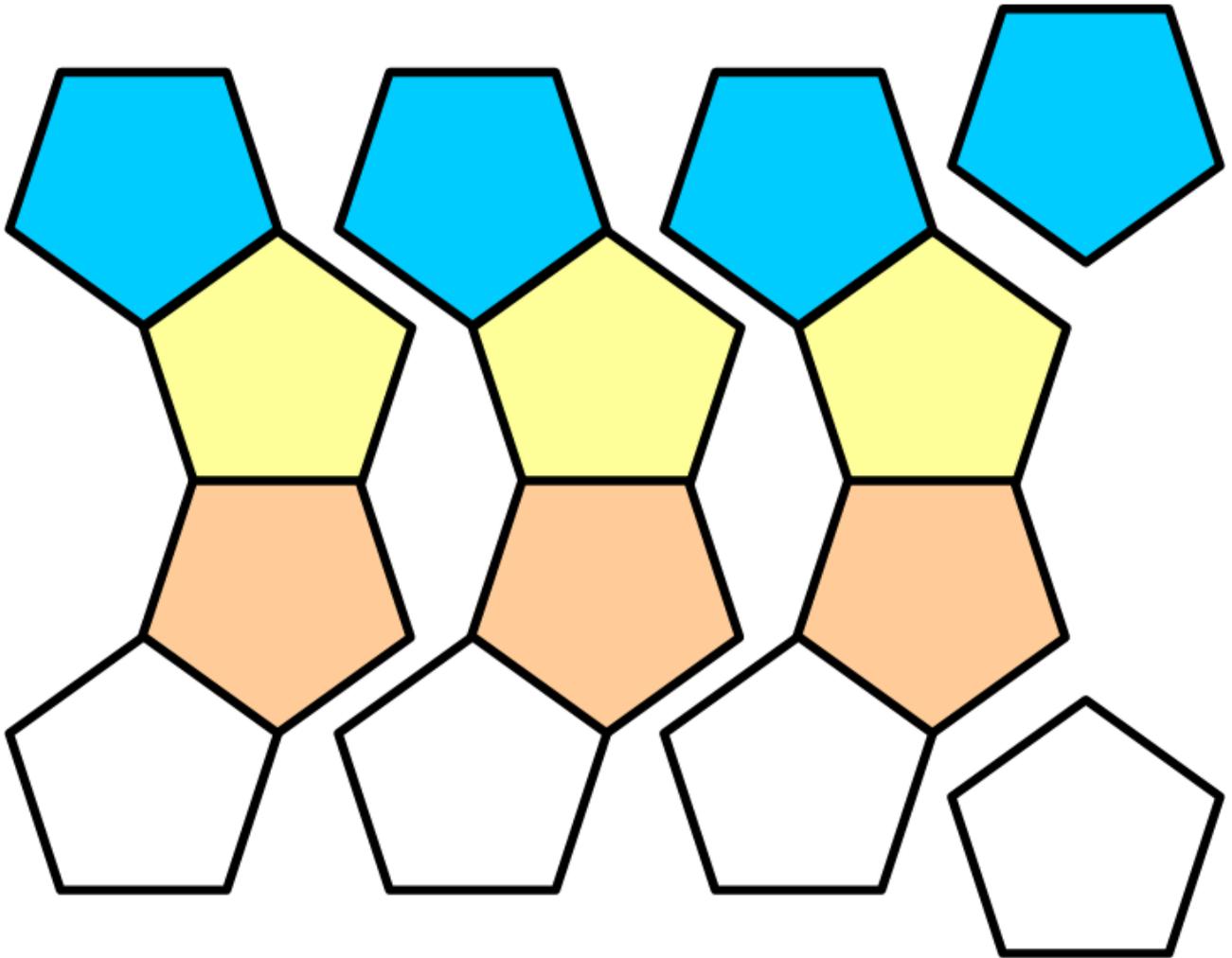
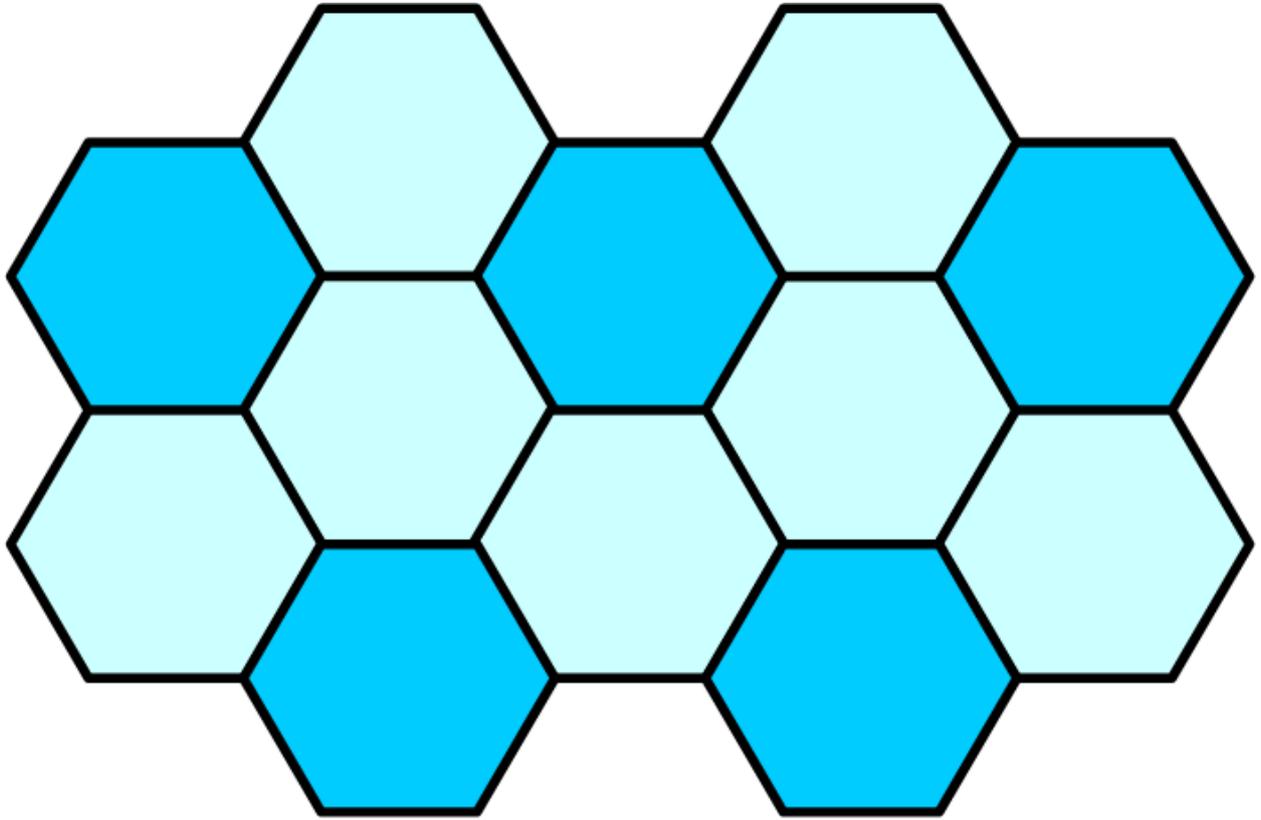
- Normal paper weight is 80gsm
- Remember to print these single-sided!

120gsm coloured paper, 10 sheets assorted colours £5.34: alexanderpapersupplies.co.uk

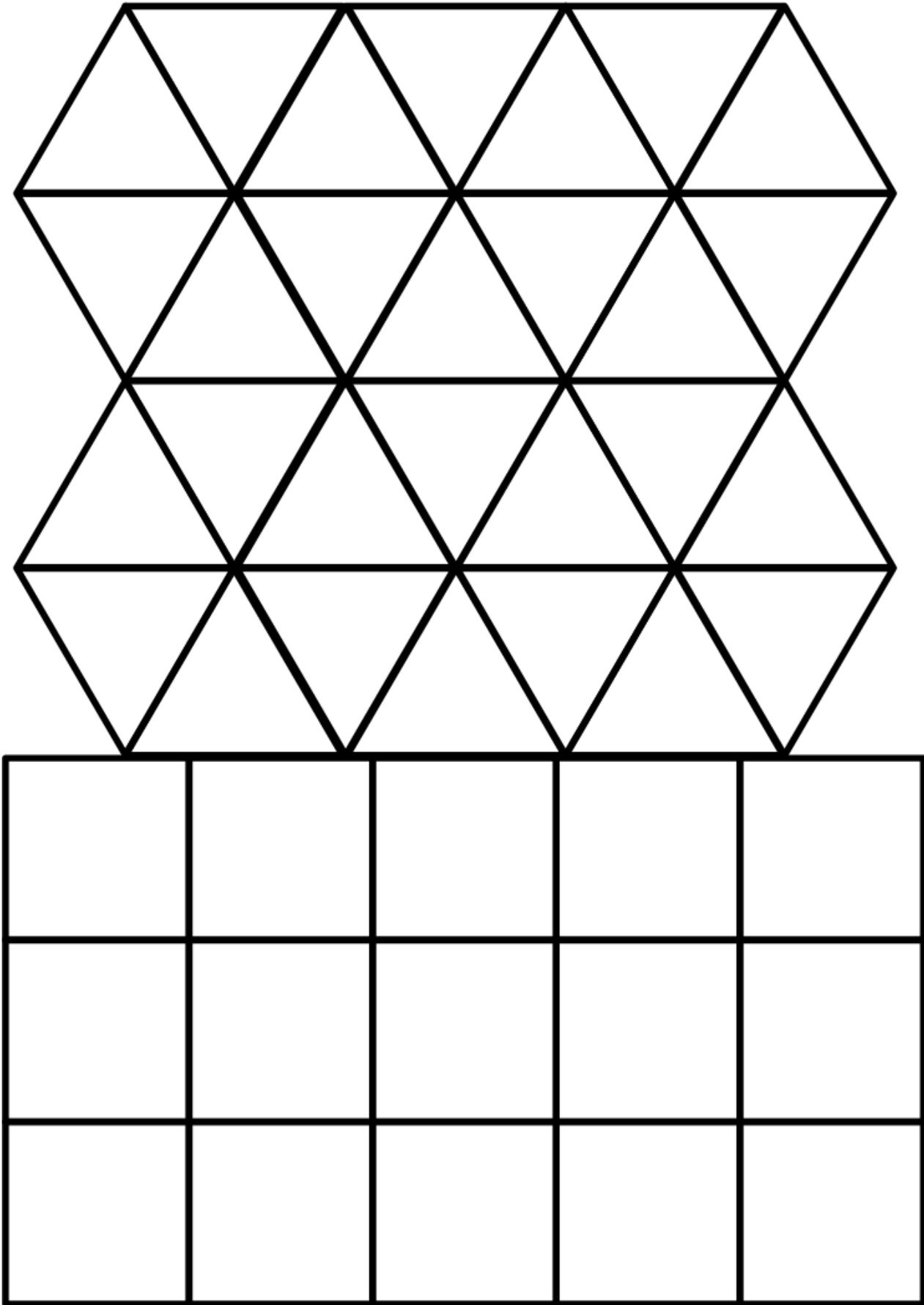
Triangle & Square Tiles: best printed on 120gsm paper. Cut into triangles and Squares before use.



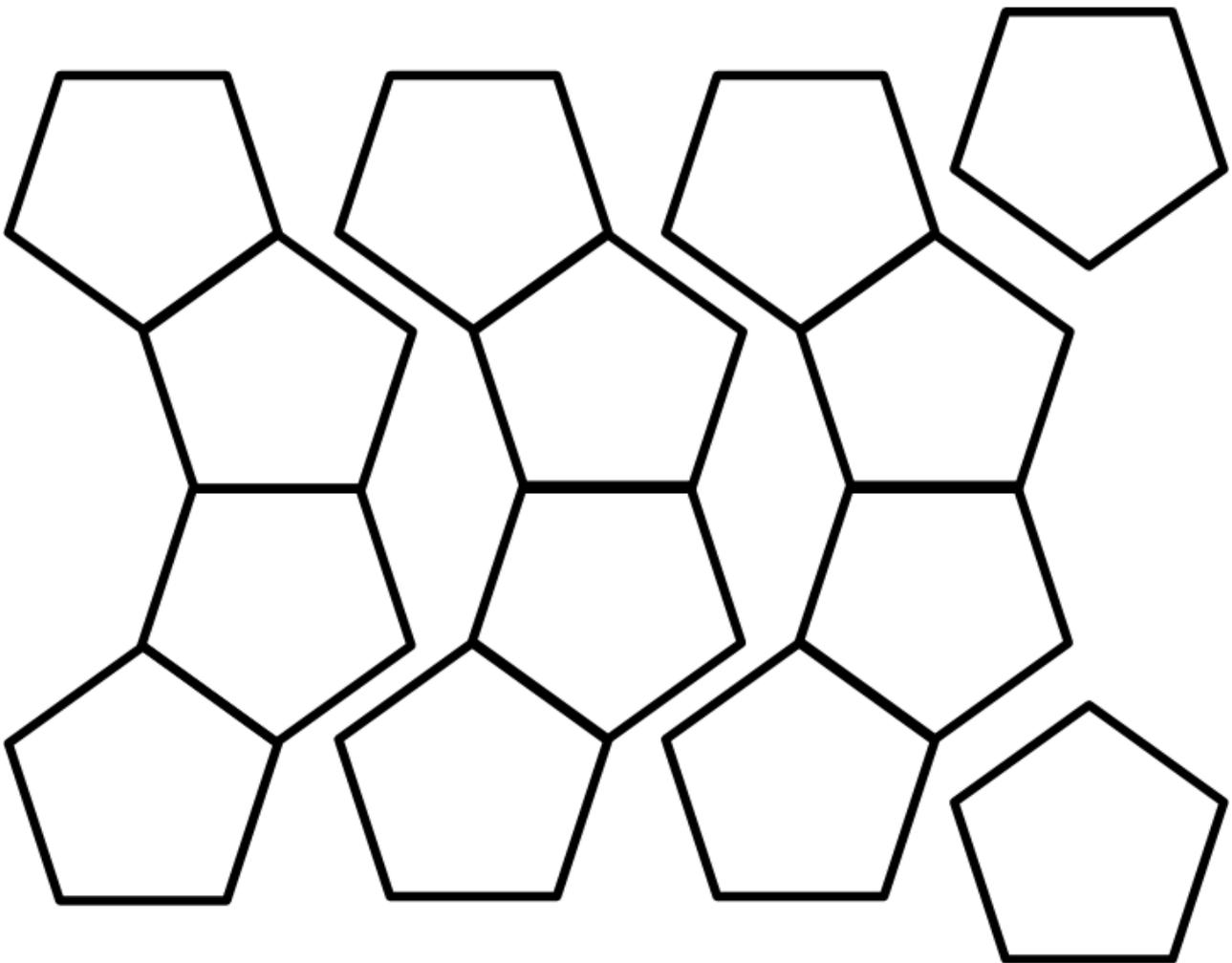
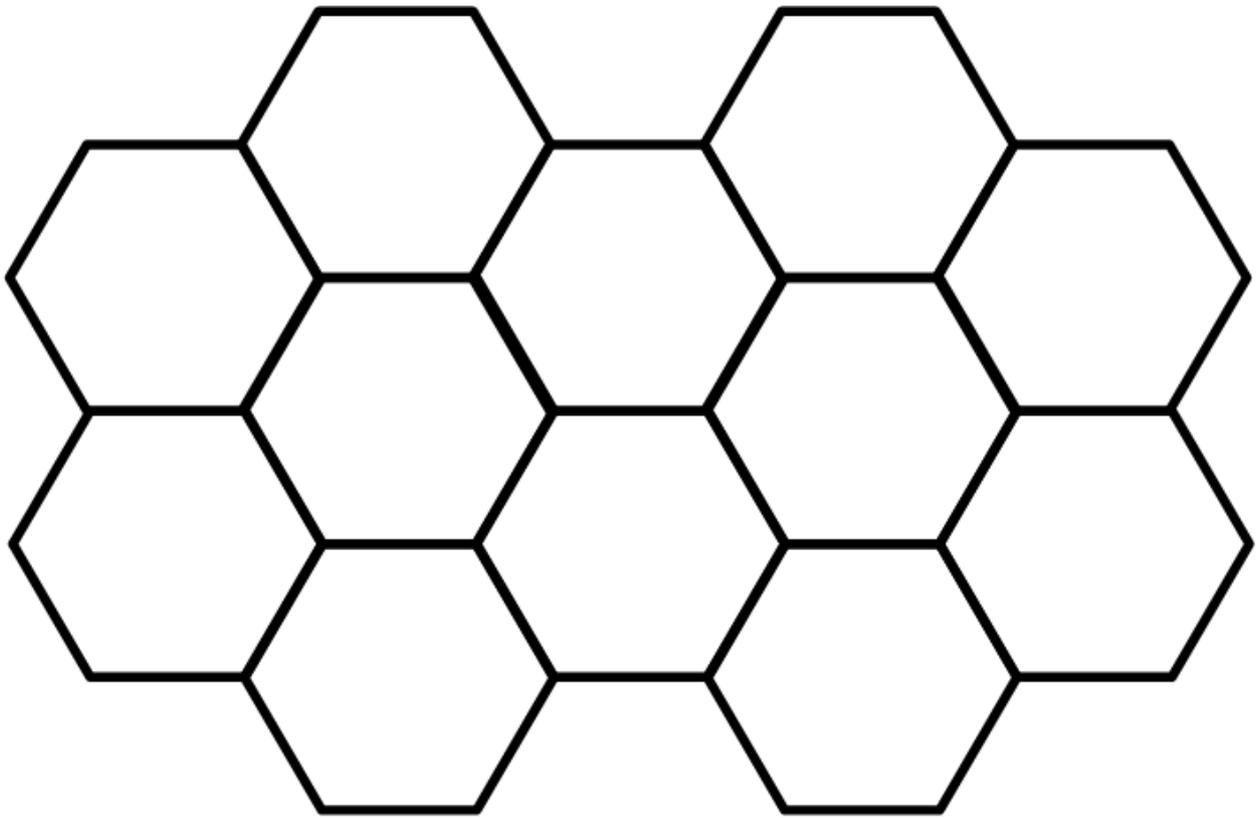
Hexagon & Pentagon Tiles: best printed on 120gsm paper. Cut into Hexagons & Pentagons before use.



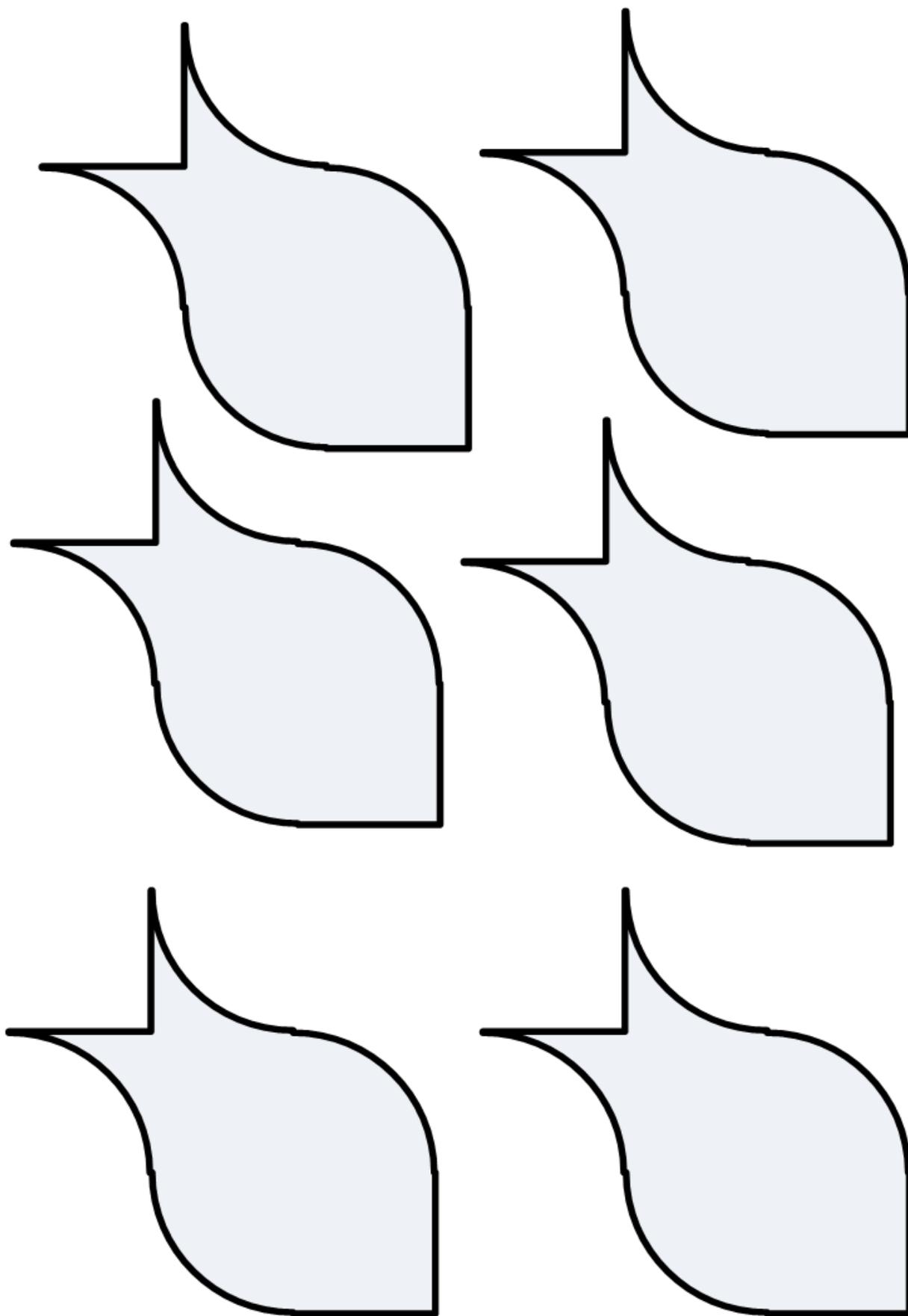
Triangle & Square Tiles: best printed on coloured paper 120gsm or heavier.



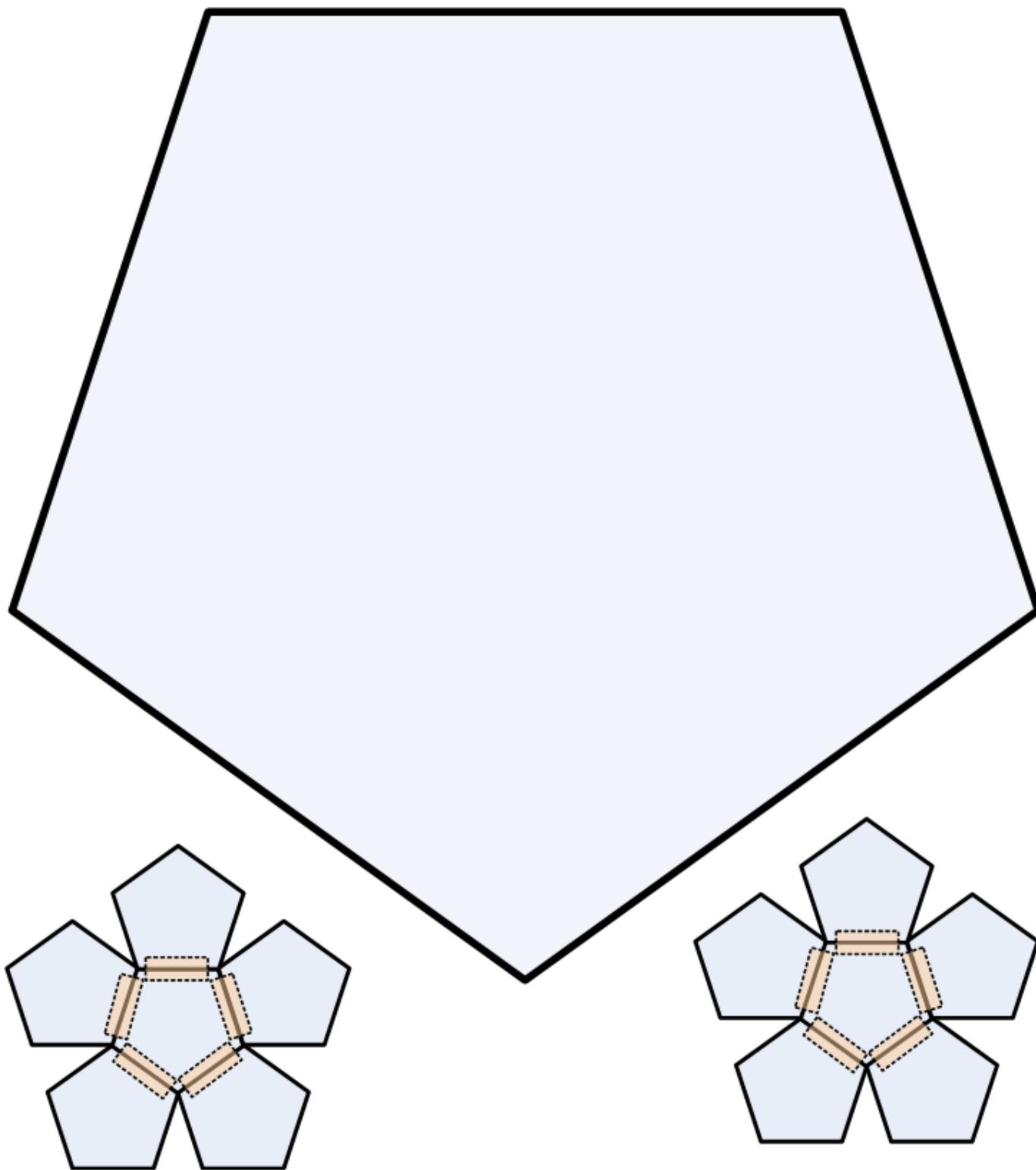
Hexagon & Pentagon Tiles: best printed on coloured paper 120gsm or heavier.



Fish Tiles: best printed on 120gsm paper. One sheet per group, for children to cut out themselves.



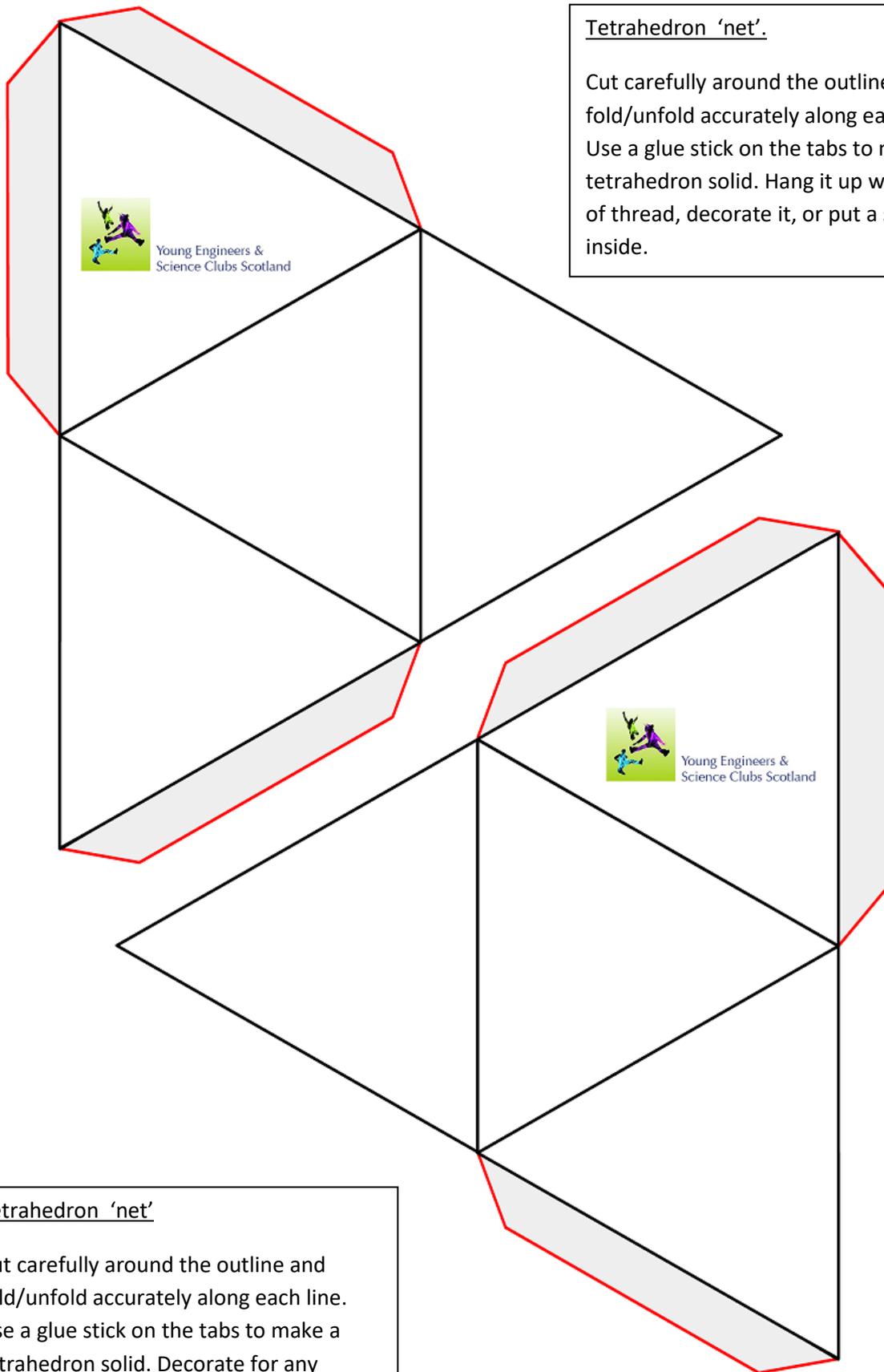
Dodecahedron Template: best cut from thick cardboard. Cut out 12 shapes and assemble with tape.



Print this template once and cut out the pentagon accurately along the inside of the black outline. Trace round this shape 12 times onto large pieces of thick cardboard (used for packing computers or washing machines). Cut out 12 identical cardboard pentagons using a sharp craft knife. Pre-assemble in two flat groups with strong tape as shown.

These two groups will fit together to make a 12-sided dodecahedron.

Tiling Extension Activity: Tetrahedron, best printed on 120gsm paper but works with 80gsm

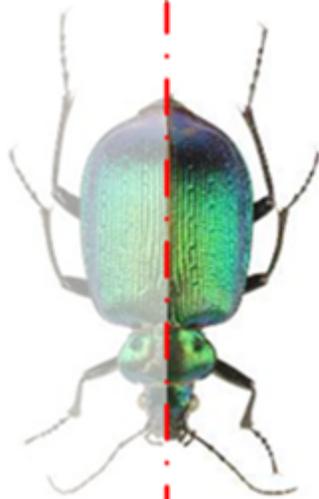


Tetrahedron 'net'

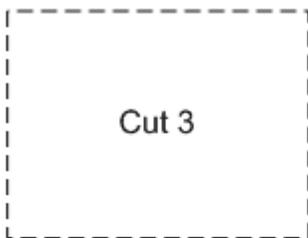
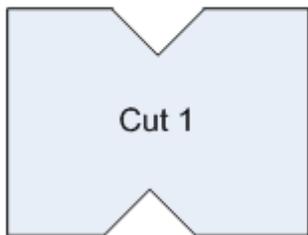
Cut carefully around the outline and fold/unfold accurately along each line. Use a glue stick on the tabs to make a tetrahedron solid. Hang it up with a piece of thread, decorate it, or put a sweetie inside.

Tetrahedron 'net'

Cut carefully around the outline and fold/unfold accurately along each line. Use a glue stick on the tabs to make a tetrahedron solid. Decorate for any theme and hang up with a piece of thread.



Fold 1



Back cover

Cut 2

Cut 1

Mysterious Maths

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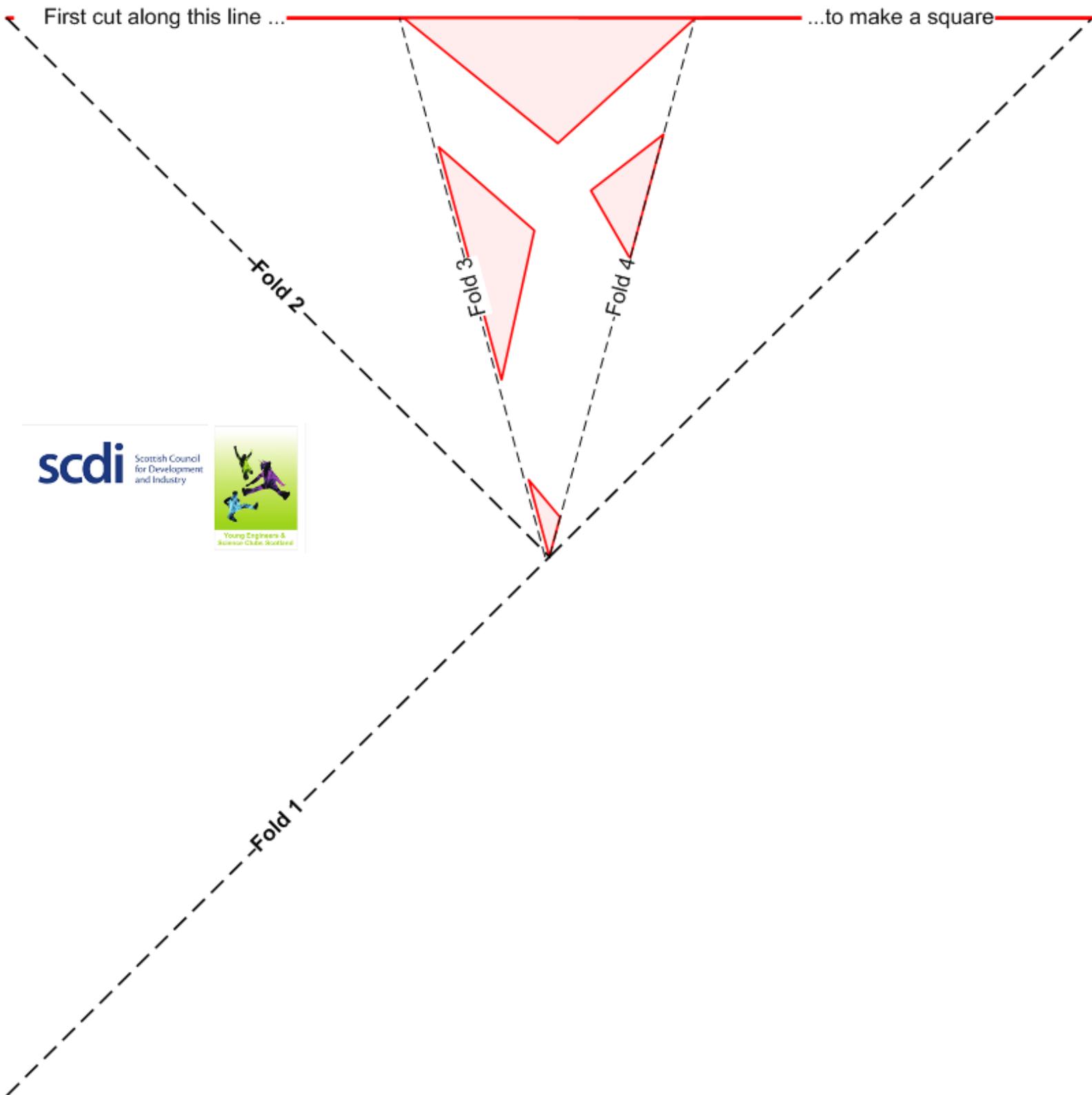
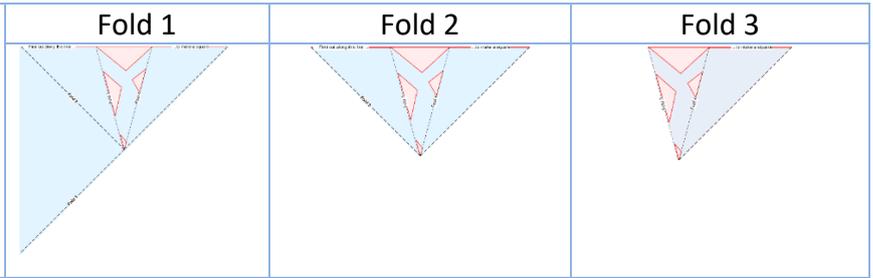
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Fold 2

Front cover

Folding Extension Activity. Thin paper (50gsm) will be easier to cut when folded.

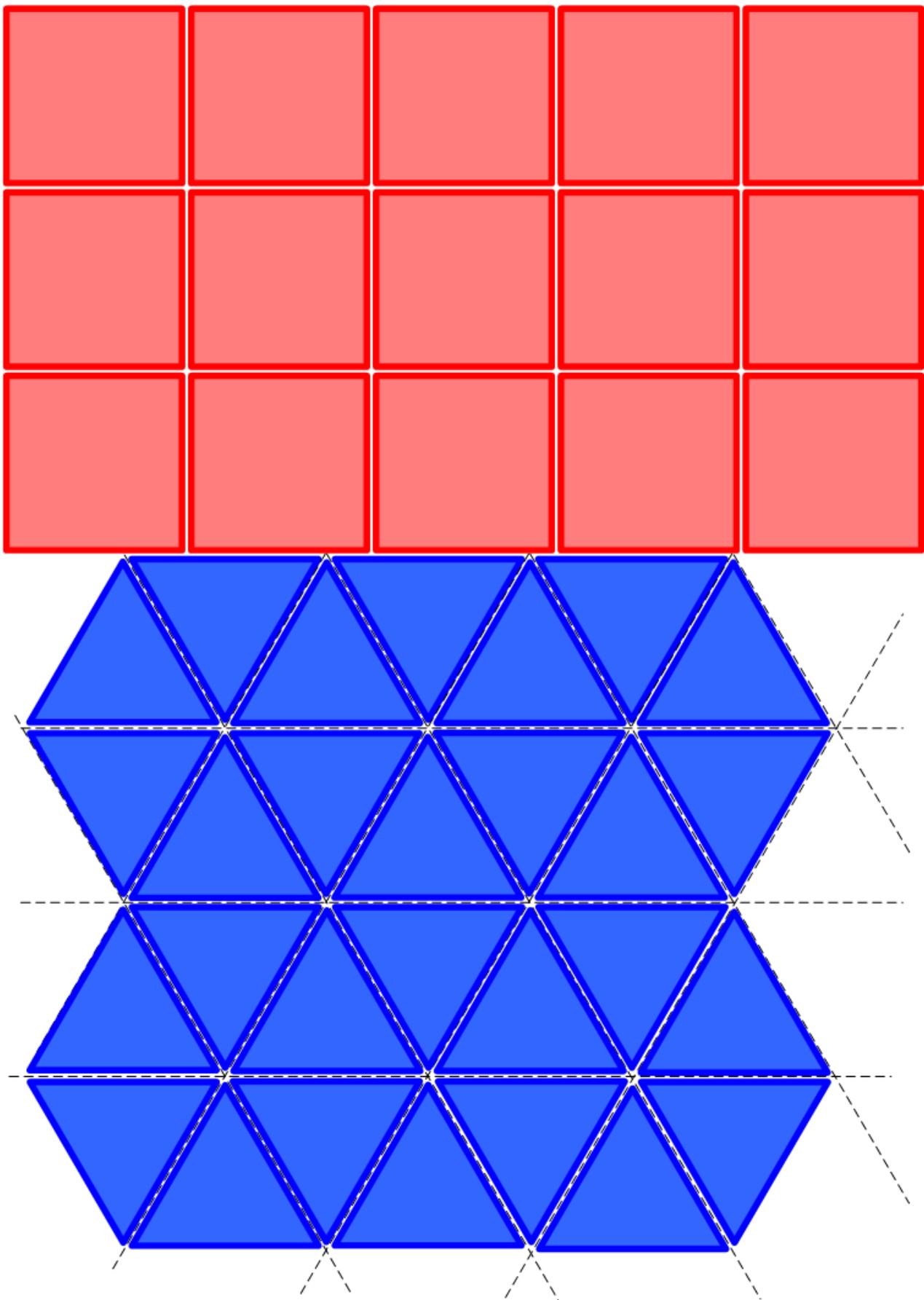
1. Cut the paper to make it square
2. Fold accurately along the black lines
3. Cut out the red triangles
4. Guess what you have made before:
5. Unfold it and find out!



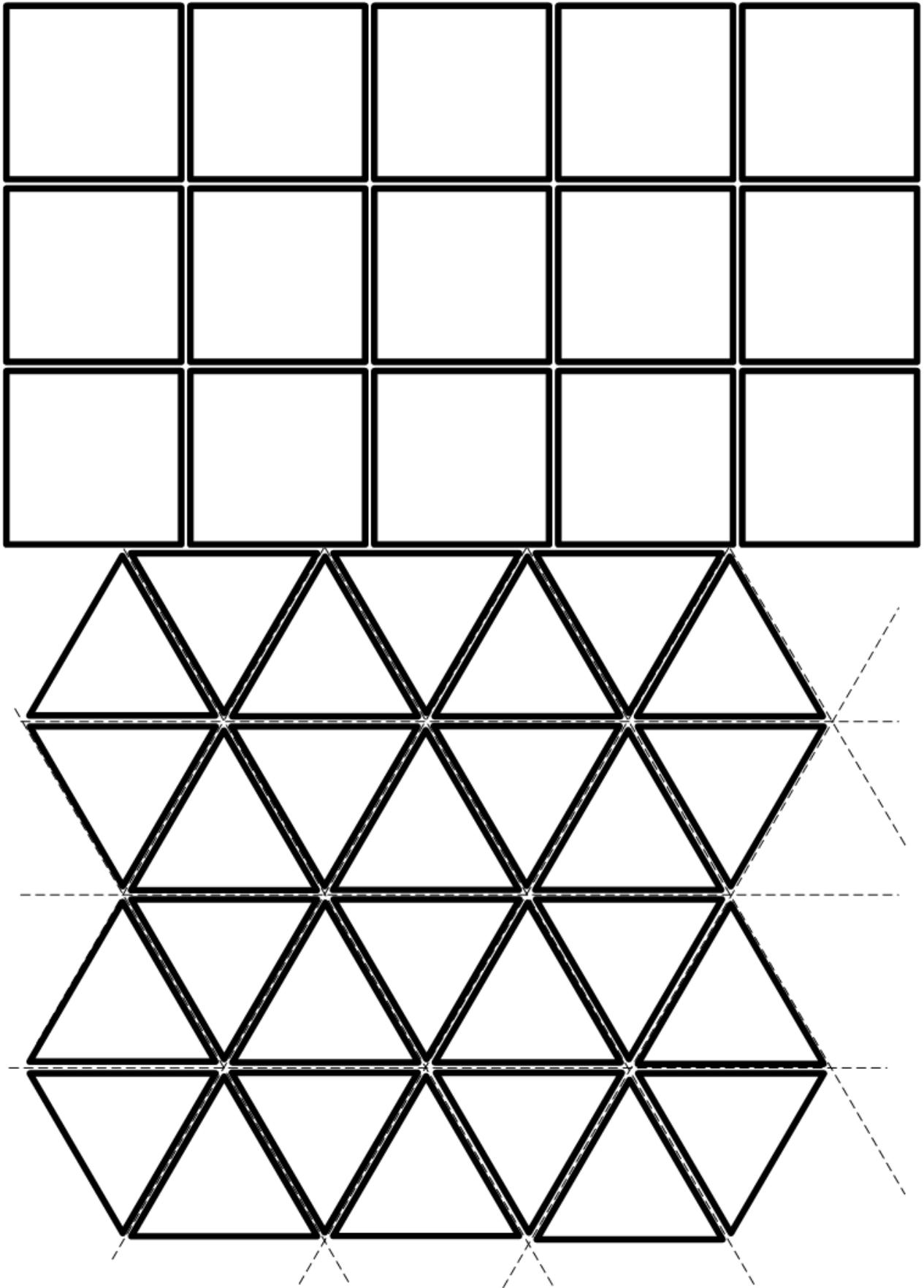
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Counters: Shapes for sticking on the Number line, for colour printing.



Counters: Shapes for sticking on the Number line, best printed on red& blue paper.

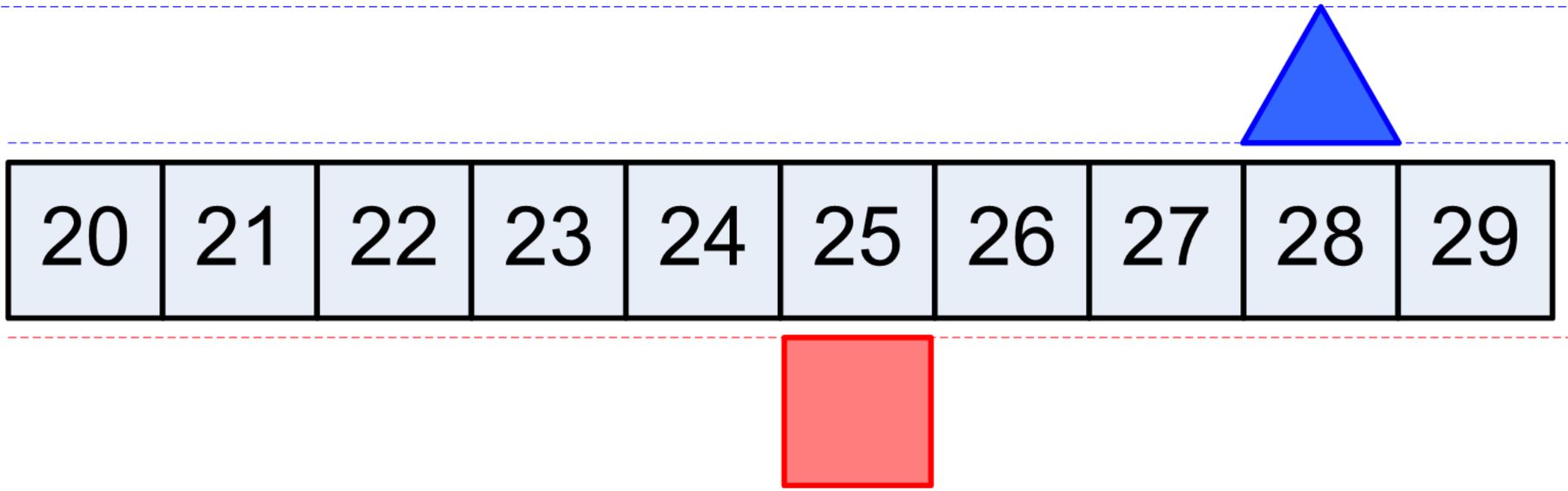


Mark Triangular numbers on this side

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

Mark Square numbers here

10	11	12	13	14	15	16	17	18	19
----	----	----	----	----	----	----	----	----	----



Mark Triangular numbers on this side

30	31	32	33	34	35	36	37	38	39
----	----	----	----	----	----	----	----	----	----

Mark Square numbers here

Mark Triangular numbers on this side

40	41	42	43	44	45	46	47	48	49
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Mark Square numbers here

Mark Triangular numbers on this side

50	51	52	53	54	55	56	57	58	59
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Mark Square numbers here

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